RAINSHADOW COMMUNITY CHARTER HIGH SCHOOL

MATH I LESSON PLAN

Teacher: Victoria M. Velazquez ***Topic***: Trig Identities

Objectives:

Students will identify which trig. Identities will be useful in simplifying trig. functions.

Students will use apply previously obtained mathematic skills to simplify trig. Functions.

Students will factor trig. Functions using trig. Identities.

Standards:

CCS: HSF-TF.C.8

Textbook Resources:

Houghton Mifflin Company. Precalculus. Larson, Ron. Hostetler, Robert. Boston, MA: 2007.

Required Materials:

Interactive Notebooks

Fodables (provided)

Worksheets (provided)

Resources List:

Pencils or pens

Notebooks

Highlighters

Tape or glue

Trig Identities

LEFT SIDE:

Page 304, Pre-Calculus text examples

Page 309: Exercises 33 –42 assignment 1

Handout last page of Lesson Plan.

RIGHT SIDE:

Trig Identities: are equalities that involve trigonometric functions and are true for every single value of the occurring variables. These functions are broken into four types: Reciprocal Identities (the inverse), Quotient Identities (tangent and cotangent Identities) , Pythagorean Identities and negative Identities. An Identity is simply when one side of the equation for an angle will equal the other side for that same angle. An identity is an equation which is always true.

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta} \quad \text{and} \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}.$$

Related identities [edit]

Dividing the Pythagorean identity by either $\cos^2\theta$ or $\sin^2\theta$ yields two other identities:

$$1 + \tan^2\theta = \sec^2\theta \quad \text{and} \quad 1 + \cot^2\theta = \csc^2\theta.$$

Negative-Angle Identities

$$\sin(-\theta) = -\sin\theta$$

$$\csc(-\theta) = -\csc\theta$$

in terms of	$\sin\theta$	
$\sin\theta =$	$\sin\theta$	$\pm\sqrt$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{}{\sqrt}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{}{\sqrt}$

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$$1 + \tan^2\theta = \sec^2\theta \quad \text{and} \quad 1 + \cot^2\theta = \csc^2\theta.$$

Each trigonometric function in terms of the other five.[2]

in terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

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in terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
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$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

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$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
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$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

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Simplify. Show all work.

1. $\cos x \sec x$

2. $\cot\theta \tan\theta$

3. $\sin^2 x(1-\cos^2 x)$

4. $\cos^4 x - 2\cos^2 x + 1$

5. $\csc^2\theta - \cot^2\theta$

6. $\cos^2\phi + \cos^2\phi \tan^2\phi$

7. $\dfrac{1+\sec x}{\tan x} - \dfrac{\tan x}{1+\sec x}$

8. $\cot\theta(\cos\theta\tan\theta + \sin\theta)$

9. $1+\cot^2\phi + \csc^2\phi$

10. $\dfrac{\sec\theta}{\cos\theta} - \sec x\cos\theta$

11. $\sin^2 x\sec^2 x - \sin^2 x$

12. $\dfrac{\sec^2 x - 1}{\sec x - 1}$

13. $\dfrac{1}{1+\sin x} - \dfrac{1}{1-\sin x}$

14. $\dfrac{\sec^4\theta - \tan^4\theta}{\sec^2\theta + \tan^2\theta}$

15. $(\sin x - \cos x)^2$

16. $(3-3\sin x)(3+3\sin x)$

Factor.

17. $\sin^2\theta - \cot^2\theta$

18. $3\sin^2\theta - 7\sin\theta - 6$

19. $\tan^3\theta - 1$

Express as a non-fractional expression and simplify:

20. $\dfrac{\cos x}{1-\sin x}$